A BAYESIAN STATISTICAL ANALYSIS OF THE ENHANCED GREENHOUSE EFFECT*

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Abstract. This paper demonstrates that there is a robust statistical relationship between the records of the global mean surface air temperature and the atmospheric concentration of carbon dioxide over the period 1870–1991. As such, the enhanced greenhouse effect is a plausible explanation for the observed global warming. Long term natural variability is another prime candidate for explaining the temperature rise of the last century. Analysis of natural variability from paleo-reconstructions, however, shows that human activity is so much more likely an explanation that the earlier conclusion is not refuted. But, even if one believes in large natural climatic variability, the odds are invariably in favour of the enhanced greenhouse effect. The above conclusions hold for a range of statistical models, including one that is capable of describing the stabilization of the global mean temperature from the 1940s to the 1970s onwards. This model is also shown to be otherwise statistically adequate. The estimated climate sensitivity is about 3.8 °C with a standard deviation of 0.9 °C, but depends slightly on which model is preferred and how much natural variability is allowed.

These estimates neglect, however, the fact that carbon dioxide is but one of a number of greenhouse gases and that sulphate aerosols may well have dampened warming. Acknowledging the fact that carbon dioxide is used as a proxy for all human induced changes in radiative forcing brings a lot of additional uncertainty. Prior knowledge on both climate sensitivity and radiative forcing is needed to say anything about the respective sizes. A fully Bayesian approach is used to combine expert knowledge with information from the observations. Prior knowledge on the climate sensitivity plays a dominant role. The data largely exclude climate sensitivity to be small, but cannot exclude climate sensitivity to be large, because of the possibility of strong negative sulphate forcing. The posterior of climate sensitivity has a strong positive skewness. Moreover, its mode (again 3.8 °C; standard deviation 2.4 °C) is higher than the best guess of the IPCC.

1. Introduction

Although voices now are raised that climate change has been detected and attributed to human interference (Santer et al., 1996), uncertainty still prevails, and this

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may easily confuse decision-makers. It is therefore worthwhile to map these uncertainties by aiming at a statistical statement on the size and the significance of the climate sensitivity, i.e., the impact of changes in radiative forcing on the global mean surface air temperature (*GMT*).

The paper starts with a brief discussion on the problem and earlier statistical approaches to it. We try to explain how much confusion there is about asking one's data the right question. Section 3 then presents the first model – almost the simplest of regression models – which indicates that the enhanced greenhouse effect is a plausible explanation for the observed global warming over the last century. Section 4 treats the most prominent alternative explanation, long term natural climate variability. It is argued that long term natural variability is an implausible explanation of the observed warming, both as a description of our data sample and from paleoclimatological evidence. Hence, consideration of long term natural variability hardly affects the confidence in the influence of the enhanced greenhouse effect. Section 5 models short term natural variability in more detail, by incorporating the influence of volcanic eruptions, El Niño, and sunspots. Yearly and even decadal deviations from the trend can be described reasonably well. Section 6 shows that the model of Section 5 does not violate its underlying assumptions. Section 7 is on the influence of non-carbon dioxide forcings. The earlier sections took the atmospheric concentration of CO_2 as a proxy for all human influences. A correction for the proxy effect is proposed. This method requires substantial prior knowledge on total radiative forcing and climate sensitivity. The priors of a number of experts are discussed, and the respective posteriors calculated. The best estimate of the climate sensitivity to radiative forcing is shown to shift upwards compared to the analysis with CO_2 only. Its uncertainty increases considerably, particularly with regard to potentially high climate sensitivities. Section 8 concludes.

2. Background

The important question whether and by how much greenhouse gases influence the climate cannot (yet) accurately be answered by general circulation models (GCMs); this is due to insufficient climatological knowledge, limited computer capacity and the large natural variability and chaotic nature of the system (Houghton et al., 1990). In addition, these models are not suited to derive a probabilistic description of key parameters, such as the climate sensitivity. Our alternative consists of sophisticatedly simple statistical models. This approach is inspired by the developments in econometrics where the failure of large models has led to a return to simplicity (Sims, 1980; Zellner, 1988).

There are statistical studies on the enhanced greenhouse effect in many kinds (see Santer et al., 1996a, for an overview). Most common is the fingerprint method (Hegerl et al., 1996; Santer et al., 1993, 1996b), which seeks an undoubtly human signal – as calculated by GCMs – in the observed record. The fingerprint approach is only applicable for detection of (dis)similarities between patterns; it seems impossible to use it to derive a probability distribution of the climate sensitivity. We use time series analysis. We do not rely on GCM results – at the expense of using an (overly) simple representation of the climate – and show that this allows to estimate a probability distribution of the climate sensitivity.

The paper has two parts, addressing two major questions. Firstly, is the observed rise in the global mean temperature ‘natural’ or ‘anthropogenic’? Secondly, if it is indeed anthropogenic, what is the climate sensitivity?

To answer the first question, we use the concentration of carbon dioxide as a proxy to all human disturbances of the atmosphere. The relevant facts are fairly simple here. The global mean temperature (source: Climate Research Unit, East Anglia) shows nonstationary behaviour, irregular but with an overall rise of 0.5 °C over the last century or so, with a slight decrease between 1940 and 1975 (cf. Figure 1). The carbon dioxide record (source: UNEP, 1990) is explosive; its natural logarithm increases linearly up to 1960 and accelerates afterwards. In time series jargon: the $\ln[CO_2]$ series is nonstationary, i.e., without fixed equilibrium level or even trending, the temperature series might be nonstationary.

Relations between nonstationary series have received ample attention in the recent econometric literature (Engle and Granger, 1987). On the one hand there is the risk of *spurious correlation:* Independent nonstationary series always seem to correlate. On the other hand, if the nonstationary character of a series (*GMT*) is due to its relationship with another nonstationary series (CO_2), this *cointegration* (Engle and Granger, 1987) leads to robust estimates of the effect. The analysis below confirms this property of robustness, i.e., insensitivity to modelling details. Galbraith and Green (1993) and Richards (1993) have applied cointegration tests to the link $[CO_2]$–*GMT*, while Woodward and Gray (1993) and Bloomfield and Nychka (1992) debate on the related question of whether the observed trend in the global mean temperature is deterministic or stochastic. The usefulness of this approach is doubted by Bayesians such as Sims and Uhlig (1991). They argue that the nature of the series (stationary, nonstationary, trending) is irrelevant in itself for statistical inference. What matters are the data that actually have been observed, and the possible explanations for their behaviour; the nature of the series then emerges as a by-product. We will work along these lines.

Earlier statistical time series modelling efforts (Gilliland, 1982; Schönwiese, 1991; Schönwiese and Stähler, 1991; Hoyt, 1979; Kuo et al., 1990) address the relationship between atmospheric carbon dioxide and climate in rather artificial ways, not explicitly testing the hypotheses of natural and anthropogenic warming to one another. In the first part of this paper, we offer a relatively simple way to incorporate doubts whether the observed warming is anthropogenic. The second part of the paper disentangles the influence of carbon dioxide from the influence of other greenhouse gases and sulphate aerosols. In both cases, the core of our analysis is Bayesian.

The Bayesian way of doing statistics is still not common ground, particularly in the natural sciences, despite tremendous progress in this field. The recent books by O'Hagan (1994) and Bernardo and Smith (1994) give a survey. The Bayesian paradigm says that probability statements – meant to take decisions under uncertainty – should be chance descriptions of the state of nature given data. Bayes' Theorem, coming from probability calculus, shows how to learn about uncertain parameters (states of nature) from the data. This requires a prior distribution, describing expert's knowledge before the data are observed. The data come in through the likelihood. Bayes' Theorem combines prior and likelihood into the posterior, the probability distribution of the parameters given the data and the prior.

The main sources of conflict between Bayesian and Classical (or frequentist) statistics are the subjective nature of the Bayesian prior, and the frequentist procedure to base statements on the probability that other observations than actually did occur would have occurred. Despite these controversies, Bayesian and Classical approaches yield similar outcomes in many cases.

The viewpoint of Sims and Uhlig (1991) above is Bayesian, the nature of a series being a typically frequentist issue. Our first analysis shows how the Bayesian analysis of the question 'natural or anthropogenic' depends slightly on prior knowledge on the possible size of the long term natural variability for the question 'natural or anthropogenic'. In the second part, separating greenhouse gases and sulphate aerosols, we have to rely heavily on prior ideas (and consequently on Bayes), as the data alone do not allow us to distinguish the respective effects.

3. Correlation and Cointegration

The observed warming is analyzed by two series models, one simple (this section and the next) and one with a number of explanatory variables (Section 5). These two models represent a whole class of models, yielding similar conclusions. Many of these models have been fitted to the data, but are not reported as they do not change the picture (cf. Tol and de Vos, 1993; Tol, 1994; Appendix A). The doubt of whether the observed warming is anthropogenic or not is introduced in Section 4. The naive version of the first model, neglecting long term natural variability, is

$$GMT_t = \underset{(2.6527)}{-28.9157} + \underset{(0.4649)}{5.0460} \ \ln[\mathrm{CO_2}]_{t-20} + \epsilon_t \tag{1a}$$

with

$$(1 - \underset{(0.0828)}{0.4156}L)\epsilon_t = u_t; \qquad \hat{\sigma} = 0.1038 \tag{1b}$$

an ARX(1)-model, fitted (in MicroTSP 7.0) to the 1870–1991 observations. Standard deviations of the parameter estimates are given in parentheses. In this model, temperature is explained by a constant, by the natural logarithm of atmospheric carbon dioxide, with a lag of twenty years and by a Gaussian AR(1)-process ϵ_t (also known as red noise), representing unexplained stationary deviations. The main criterion to judge the model quality is $\hat{\sigma}$: the estimated standard deviation of the one step ahead forecast error; other criteria, such as R^2 or the loglikelihood, are monotone transformations of $\hat{\sigma}$. The twenty-year lag stems from the theoretically expected slow response of the *GMT* to changes in atmospheric CO_2 concentration. A similar specification is used by Schönwiese (1991). In subsequent models we use 'distributed lags', which is more elegant but does not change the conclusions. The data mildly support the idea of a slow response. In fact, atmospheric carbon dioxide rises so smoothly that the data have very little power in distinguishing between different lags and lag structures. Radiative forcing due to changes in the atmospheric concentration of carbon dioxide are known to be approximately proportional to the natural logarithm of the concentration (Shine et al., 1990).

The simple model (1) appears to be the best amongst a number of alternatives. Different lags for the influence of carbon dioxide lead to lower values of $\hat{\sigma}$, without changing the conclusions regarding the climate sensitivity (Tol and de Vos, 1993). Table A-I (Appendix A) shows that the main result of model (1) is also robust to the choice of error structure. The estimated parameter for the influence of carbon dioxide on the global mean temperature hardly differs whether an AR(1) structure is chosen, an ARMA(4,1), or anything in between; it is highly significant in all cases. So, in line with cointegration theory, in any model similar to (1), we find a similar, significant effect of CO_2. This tentatively shows that the global mean temperature and carbon dioxide records correlate and cointegrate. In addition, carbon dioxide clearly leads temperature. Moreover, the assumptions of normality and homoscedasticity for the residuals of (1) are not rejected. Had there not been alternative hypotheses to explain the climate record, this would have been sufficient to establish detection and attribution of human induced climate change. However, alternative explanations do exist.

4. Long-Term Natural Variability

A more severe test for model (1) is to include a linear trend (t) to account for a possible 'spontaneous' long-term rise in the temperature – this is a catch-all for other potential causes of global warming (see Appendix A for non-linear trends). We get the following result (t scaled such that its coefficient represents the change in a century):

$$\underset{}{GMT_t} = \underset{(12.2714)}{-35.0133} + \underset{(2.1990)}{6.1398} \ln[CO_2]_{t-20} - \underset{(0.2161)}{0.1098}t + \epsilon_t \tag{1′a}$$

with

$$(1 - \underset{(0.0836)}{0.4101}L)\epsilon_t = u_t; \qquad \hat{\sigma} = 0.1042 \tag{1′b}$$

Model ($1'$) is almost identical to model (1) but for the t-value of the [CO_2] coefficient which drops to 2.79, still significant at the 95% level but considerably less convincing. The message is clear: $\ln[CO_2]$ explains the rise in *GMT* much better than a linear trend. The Classical statistical procedure is to test the significance of the trend. This would lead to a rejection, and a return to (1). The Bayesian alternative is to formulate priors on the parameters. In this view, outcome ($1'$) corresponds to absolute a priori uncertainty about the parameters (non-informative priors), while (1) assumes that we know a priori for certain that the parameter for the trend is zero. Other priors give results in between. Technically, a prior can also be incorporated via 'mixed estimation' (Theil and Goldberger, 1961; cf. Appendix B). Note that we only use a prior on the trend; implicitly, non-informative priors are used for the other parameters (see further Section 7).

Prior knowledge is obtained from a *GMT* record for the 10,000 years preceding our sample (Houghton et al., 1990, page 202, middle figure). This record may be seen as a series of 100 observations of temperature changes over a century. The forecast, and its associated uncertainty, of the natural temperature change for this century from this record, is the prior that we need. The 'prior *GMT*' record shows that long periods with considerable changes in temperature have occurred, but also that these changes develop gradually which 'justifies' the use of a linear trend in ($1'$); cf. Appendix A. The large and abrupt changes in temperature (Dansgaard et al., 1993; GRIP Members, 1993) which have drawn a lot of attention are, so far, largely regional events with clear triggers, such as a collapse of Greenland's ice-sheet (Lehman, 1993), which have not been observed recently (see Nicholls et al., 1996, for an overview). Fitting an AR(2) model to the 'prior' *GMT* record predicts a rise of 0.01 °C in the 20th century, with a standard error of 0.12 °C. Alternative models yield similar results. Thus, a rise of more than 0.25 °C per century is a priori implausible (its chance is less than 5%). Re-estimating ($1'$) with the corresponding prior results in virtually the same estimates for the [CO_2] parameter, but its t-statistic rises to 4.58. As a sensitivity analysis on the prior derived from Houghton et al., we double the standard deviation to 0.24 °C/century.* A t-value of 3.37 results. This is still considerably more significant than the 2.79 of ($1'$) which corresponds to an infinitely large prior standard error (i.e., a non-informative prior). See Table I.

For non-linear specifications of long-term natural variability, the conclusion that carbon dioxide has a significant influence on the temperature is not affected (Appendix A). The role of the prior would be more important, as the implied natural temperature change would be larger (cf. Table A-II). However, not even with cubic long-term natural variability the prior is decisive for the influence of carbon dioxide to deviate significantly from zero. In addition, we argue in Section 7 that the uncertainty about the long-term natural variability is dominated by the uncertainty about the radiative forcing in the twentieth century.

* Wigley and Kelly (1990; cf. also Nicholls et al., 1996) consider it unlikely that global mean temperatures have varied by 1 °C or more in a century at any time during the last 100 centuries; according to this prior, the chance of this is about 0.3%.

In Appendix A, a random walk with drift is explored as another alternative for the enhanced greenhouse effect. In this model, a linear trend is combined with a non-stationary error specification. The coefficient of ln[CO2] and its significance are affected, but the model is outperformed by model (1′); see Table A-III. In addition, a large long-term natural variability results if carbon dioxide is left out.

In sum, it appears that the influence of atmospheric concentration of carbon dioxide on the global mean surface air temperature is robust against a number of representations of the alternative hypothesis of long-term natural variability. The confidence in the estimate of the climate sensitivity is lower than in the case that the alternative of natural variability is ignored, but the finding that the enhanced greenhouse effect is the most plausible explanation of the observed global warming stays intact.

5. Short-Term Natural Variability

If the simple models of the above are replaced by a more sophisticated one, the message that the influence of the enhanced greenhouse effect on the *GMT* is real is further confirmed. We replace the twenty-year lag of ln[CO_2] by a 'distributed lag' (a second-order Almon (1962) lag with 40 lags and zero restrictions at both sides; see Appendix B), representing a gradual effect on *GMT*. A couple of other explanatory variables of the global mean temperature are also taken up. These are the dust veil index (*DVI*) for the volcanic activity (Lamb, 1977), the sunspot numbers (*SSN*) for the solar activity (Waldmeier, 1961) (both are divided by one thousand so as to get the parameters in a proper range) and the southern oscillation index (*ENSO*; Lamb, 1977). These indices do not raise problems with respect to multicollinearity.* Lagged *GMT* captures the first-order autocorrelation, so the noise u can be white. This requires some rescaling of the direct regression results, which is done in such a way that the coefficients of [CO_2] and t (with an a priori standard error of 0.12 °C/century) may directly be compared to the previous results. The regression results are:

$$\begin{aligned} GMT_t \;=\; & \underset{(5.1794)}{-17.9785} + \underset{(0.0780)}{0.4309}\, GMT_{t-1} + \underset{(1.5179)}{5.5317}(1-0.4309)\ln[CO_2]_{ALM(40,2)} \\ & + \underset{(0.1858)}{0.3792} SSN_{t-1} - \underset{(0.0329)}{0.0407} DVI_t - \underset{(0.0372)}{0.1182} DVI_{t-1} - \underset{(0.0372)}{0.0998} DVI_{t-2} \\ & - \underset{(0.0352)}{0.0619} ENSO_t - \underset{(0.0116)}{0.0332} ENSO_{t-1} - \underset{(0.0841)}{0.0386}\, t + u_t\,, \\ \hat{\sigma} \;=\; & 0.0879. \end{aligned} \tag{2}$$

* Maddison (1994) takes a different position. He argues that, since the distributed lag represents the climate inertia to an initial shock, the lag structures of carbon dioxide, volcanic eruptions, ENSO and sunspots should be identical. However, we prefer to interpret the lag structure as a chain of positive and negative feedback mechanisms set in motion by the enhanced greenhouse effect. As such, our model is explicitly interpreted as a local approximation to the full system, whereas Maddison is more confident in the agreement between reality and model.

The residual standard error is considerably lower due to the significant contributions of the added explanatory variables, which are plausible in sign.

Tol (1994) and Tol and de Vos (1994) further discuss the influence of the short-term natural variability explanatories on the observed *GMT*, accounting for the observed dip between 1945 and 1970. The first paper also discusses the influence of the sunspots, on which there is some discussion whether or not it is stationary. It rejects the hypothesis that it is the sun rather than the enhanced greenhouse effect inducing global warming.

6. Discussion

Model (2) is extensively tested for deviations from its underlying assumptions, including non-normality (Jarque and Bera, 1980), serial correlation (Ljung and Box, 1978), heteroscedasticity (Harvey, 1989) and non-linearity (McLeod and Li, 1983), and found to perform well. The RESET test (Ramsey, 1969) is not passed, however, which may point to some misspecification. This little flaw is more than compensated by another result. Re-estimating the model for the period up to 1940 and 'forecasting' the remaining 51 observations (conditional on the exogenous variables, but not using *GMT* observations after 1940) leads to Figure 1. The overall quality of the 'forecast' is quite remarkable for a time-series model like this and restores the confidence in the parameter stability and other aspects of the model.

The results on the [CO_2] coefficient depend, like before, on the prior for the trend coefficient. Table I contains the outcomes for various priors on the long-term natural variability, together with those for the ARX(1) model. All coefficients and standard errors are multiplied by ln 2 in order to obtain the effect of a doubling of CO_2. The conclusions of model (2) are stronger, due to the better description of the temperature. Noteworthy is that in the general model the greenhouse effect is also significant at the 99% level for $\sigma_\beta = \infty$ (i.e., unrestricted inclusion of the trend). Furthermore, it strikes that the size of the estimates of the climate sensitivity hardly differs between the ARX(1) model and model (2). This is in line with the literature on cointegration: For inference on relations between nonstationary series, extension of a model with stationary explanatory variables hardly affects the conclusions.

From this point of view, Table I may be considered to represent the extreme cases of a wide range of models. The ARX(1) model represents abstinence of modelling details, model (2) belief in (our) detailed explanation, thereby spanning a substantial part of the model space. A balanced statement on the impact of the enhanced greenhouse effect, discounting for 'data mining' in model (2), would end up somewhere between these two. Likewise, moving from the left to the right of the table represents decreasing belief in the long-term natural variability, with a balanced judgement somewhere in the middle, say a best estimate of 3.8 °C with a standard deviation of 0.9 °C.

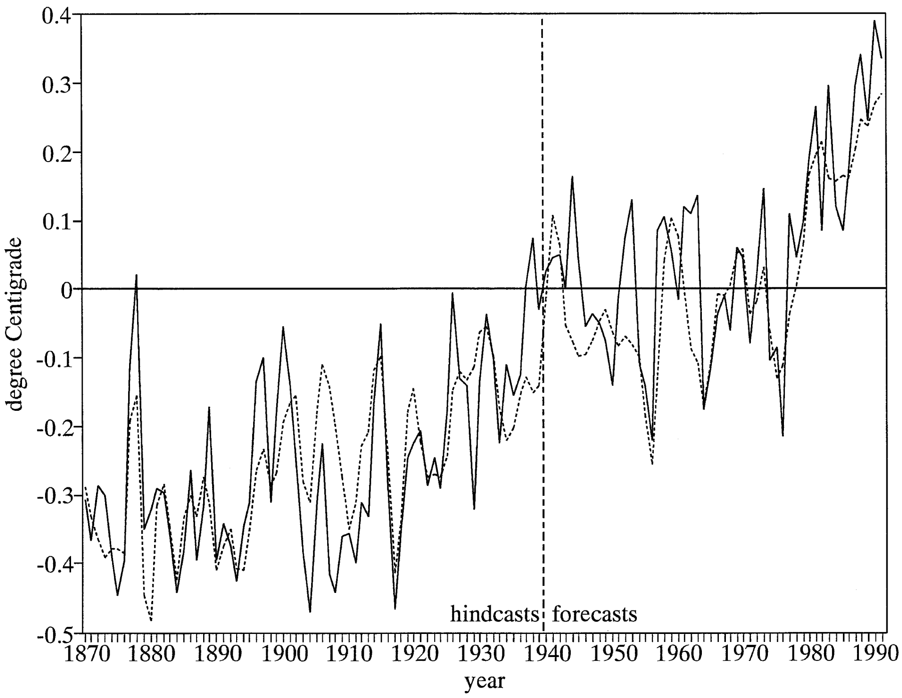

Figure 1. Annual global mean surface air temperature (in °C) as observed (solid line) and as modelled (dotted line) by Equation (2), without trend. The period 1870–1940 (hindcasts) is used to estimate the parameters, the period 1941–1991 (forecasts) is used to validate the model.

Table I
Equilibrium temperature change (°C) at 2 × $[CO_2]$[a]

Model	$\sigma_\beta = \infty$	$\sigma_\beta = 0.24$	$\sigma_\beta = 0.12$	$\beta = 0$
ARX(1)	4.2558	3.8860	3.6245	3.4976
(1)–(1′)	(1.5243)	(1.1528)	(0.7912)	(0.3222)
General	4.3952	4.1730	3.8343	3.3701
(2)	(1.4505)	(1.3073)	(1.0521)	(0.2855)

[a] β denotes the parameter of the long term natural variability, σ_β is its prior standard deviation.

7. Other Forcings

It requires some care to state what we may conclude from the analysis above. This is due to the fact that the atmospheric concentration of carbon dioxide is only a proxy for the set of human disturbances of the atmosphere. We may therefore not conclude that a doubling of atmospheric CO_2 will lead to a temperature rise of $(3.8 \pm 2 \cdot 0.9)$ °C. Ignoring explanatory variables which correlate with the CO_2 record affects both the mean and variance of the parameter (in Classical terms:

the estimate of the parameter and its confidence). Over the last century, a number of causes for climate change can be identified. Not only has the atmospheric concentration of carbon dioxide changed, but also the atmospheric concentrations of methane, nitrous oxide, ozone, halocarbons, and sulphate aerosols.

The models summarized in Table I lump all these influences together, and assume that the composite can be described by carbon dioxide. Given shortage of data, and given the fact that these influences run largely parallel to carbon dioxide, there is little alternative. It does imply that, unless all other positive and negative forcings happen to cancel out, our estimate does not apply to the climate sensitivity. Houghton et al. (1994) give 1.5 W/m^2 as best guess for the change in radiative forcing due to carbon dioxide since preindustrial times, whereas total forcing changed about 1.1 W/m^2 over the last century. Thus, to estimate the climate sensitivity we should multiply the earlier estimate by a factor of 1.5/1.1. In addition, knowledge on changes in radiative forcing over the last century is far from precise. By assuming that forcing is perfectly known, the range of the climate sensitivity is overconfidently narrow.

Because of lack of data and multicollinearity between the various greenhouse gases and sulphate aerosols, we assume their forcing profile over time to be equal to the forcing by carbon dioxide (i.e., ln[CO_2]). Statistically, this is the worst case: the respective effects cannot be distinguished on the basis of the observations. Let F_t denote total radiative forcing at time t, then

$$F_t \approx c_1 \ln[\mathrm{CO_2}]_t + c_2\sqrt{[\mathrm{CH_4}]_t} + c_3 f([\mathrm{aerosols}]_t) + \ldots := \omega \ln[\mathrm{CO_2}]_t \qquad (3)$$

(see Houghton et al., 1994). Equation (3) assumes that the development of total radiative forcing is proportional to radiative forcing due to carbon dioxide. Scaling parameter ω is a measure for the relative contribution of CO_2 (1/ω is the share of CO_2).

Let α denote the parameter of the (Almon-transformed) carbon dioxide concentration in Equation (2). Then, the equilibrium reaction of the global mean temperature to a permanent change in the atmospheric CO_2 is $\alpha/(1-\phi)$, where ϕ is the parameter of the lagged temperature in (2). Let ζ be the parameter of interest, i.e., the influence of radiative forcing on temperature,* then

$$\zeta\omega = \frac{\alpha}{1-\phi}\,. \qquad (4)$$

We have information on α and ϕ, and thus on $\alpha/(1-\phi)$, from the analyses above, but we are looking for information on ζ. It is clear from Equation (4) that the estimate of ζ, climate sensitivity, depends on the assumptions one makes on ω, radiative forcing. Indeed, in the parameterization adopted here, radiative forcing and climate

* The parameters c_i of Equation (3) are linear functions of ζ. Both c_i and ζ denote radiative forcing, but ζ relates to radiative forcing for whatever cause, whereas c_i relates to radiative forcing due to a specific gas.

sensitivity are unidentifiable. That is, the data cannot distinguish between the two. Additional information is needed.

From here on, the inference is fully Bayesian. It must be, as without prior information no progress can be made. Despite a number of complications, the Bayesian analysis below can be reduced to manipulations that can readily be reproduced and understood. The first point is that the prior information on β, the parameter for the long-term natural variability, is incorporated in our estimates of α and ϕ, so that we can proceed where we ended in the last section. The proof is in Appendix B. For brevity's sake, only results with the more conservative prior ($\sigma_\beta = 0.24\,^\circ$C per century) are reported; the outcomes are not strongly influenced by this.

The posterior distribution of ζ and ω follows from Bayes' rule,

$$p(\zeta,\ \omega|\text{data}) \propto \pi(\zeta,\ \omega)L(\text{data}\,|\zeta\omega). \tag{5}$$

Equation (5) can readily be evaluated numerically. The likelihood is the same for all points with the same value for $\zeta\omega$. This is no problem as long as proper priors for ϕ and ω are used. The marginal posteriors follow by integration. The choice of prior is crucial. Six couples of priors are used, spanning a wide range of opinions on human induced climate change. First, Figure 2 shows the distribution of $\alpha/(1-\phi)$, which is that of ζ if ω is assumed to be unity with certainty (as we implicitly did in the above). The left-skew of the density is a result of the negative correlation between the estimates of α and ϕ.

The first two priors on climate sensitivity and radiative forcing are uniform, almost non-informative ones; for ζ, the lower and upper bounds are 0 °C and 25 °C; for ω, the lower and upper bounds are 0 and 1.4. Figure 3 displays the results (note that the change in radiative forcing between 1870 and 1991 is displayed instead of ω). Both posteriors have the same, strongly right-skewed form; the similarity follows from the fact that the respective influences are unidentifiable. Table II summarizes the characteristics of the posteriors. The remaining five posteriors of ζ are combined with a triangular (0, 0.7, 1.4) prior on ω, conform Houghton et al. (1994). Firstly, the triangular prior on ω is combined with a uniform (0, 25) prior on ζ. The posterior on the climate sensitivity keeps its shape, but the right tail is less pronounced. The data push the posterior for radiative forcing a bit downward compared to the prior.

Because we use the same prior on ω in combination with the experts' priors on ζ below, and because the uniform prior on ζ above is approximately non-informative, the posterior on ζ above is very close to the likelihood appropriately marginalized on ω (see Appendix B). This means that one can obtain the posterior of climate sensitivity by multiplication of the ordinates of the prior with those of the 'likelihood' given by Figure 2c.

The most obvious prior for the climate sensitivity is the IPCC's (Houghton et al., 1990, 1992, see also 1996). The range 1.5 to 4.5 °C equilibrium temperature rise for a doubling of atmospheric carbon dioxide is assumed to be a 75% con-

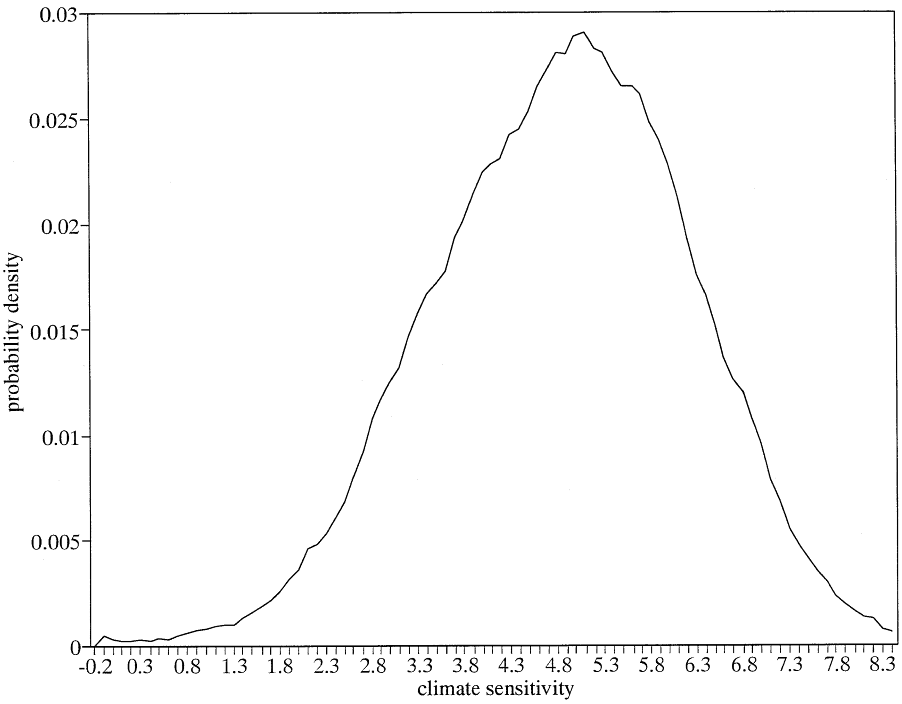

Figure 2. Probability density of $\alpha/(1-\phi)$, with α and ϕ as estimated with model (2) for σ_β = 0.24 °C/century.

fidence interval. The prior is assumed to be triangular. A triangular (0.14, 2.5, 6.22) prior results. Figure 3 displays the posterior. Again, low climate sensitivity is excluded by the data, though not completely, because we include long-term natural variability. Radiative forcing assumes a similar shape. The IPCC's assessment is the result of consensus building. Morgan and Keith (1995) report a formal expert elicitation, including fourteen well-known U.S. climatologists. We choose three of them here, and fit a Gumbel prior to the experts' expectation and standard deviation, as most experts point at a right skewed prior, and do not want to exclude greenhouse gas induced global cooling. Expert number 4 is a very uncertain man (the experts remained individually anonymous but are known as a group; no women were involved). Allowing for surprises, the climate sensitivity could be anywhere between –10 °C and 20 °C, with a mean of 4.7 °C and a standard deviation of 5.4 °C. The likelihood shows that this range is too broad. The prior chance of a climate sensitivity greater than zero is 0.824; the posterior chance is 0.999. Little information is added on radiative forcing, although the posterior is a bit lower than the IPCC (Houghton et al., 1994) suggests in order to allow for the high climate sensitivities. See Figure 3 and Table II.

Expert number 5 is the opposite of expert number 4. He judges climate sensitivity to be very small, only 0.3 °C, and is rather certain of himself as his standard

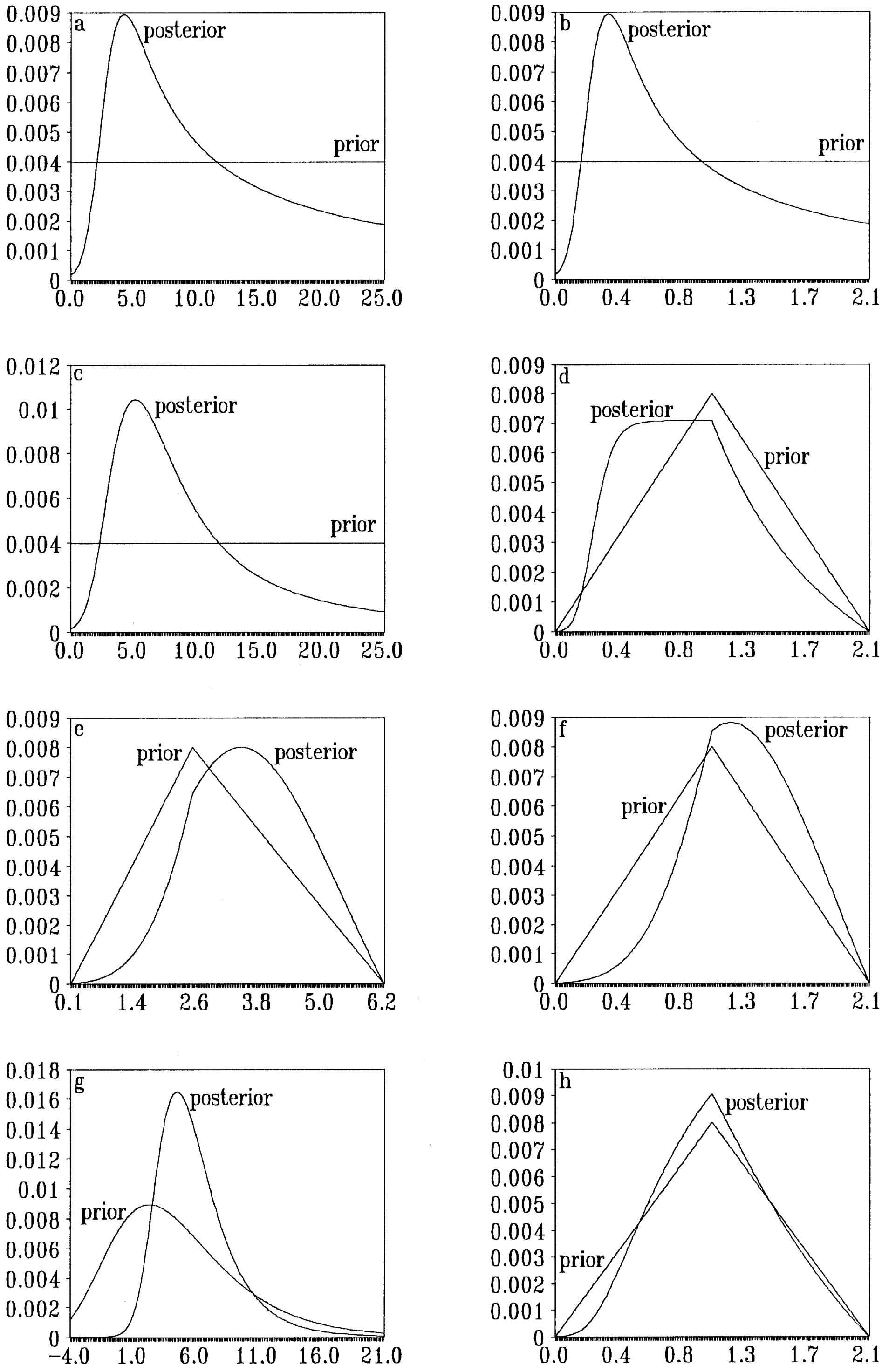
a
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Figure 3a–h.

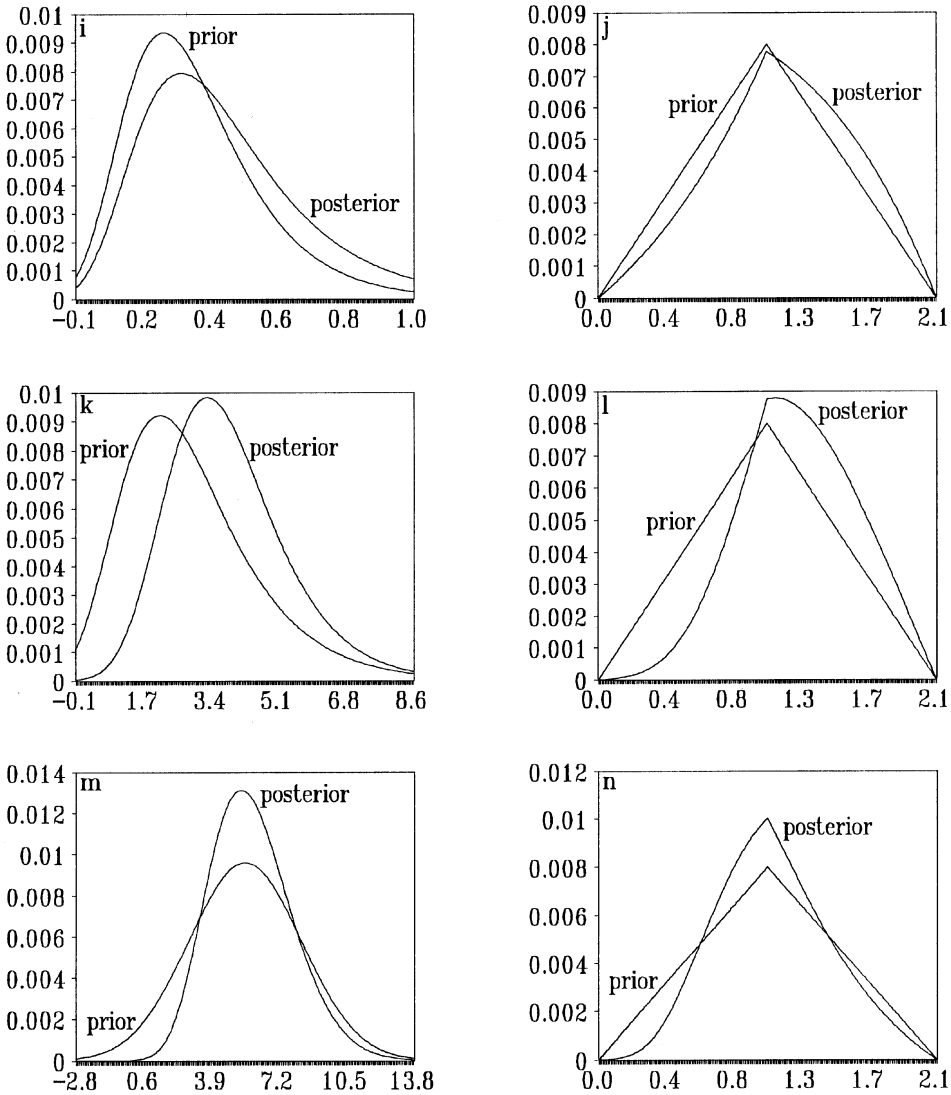


Figure 3i–n.

Figure 3. Prior and posterior probability density functions of the climate sensitivity to a doubling of atmospheric carbon dioxide equivalents (in °C; left panel) and radiative forcing since pre-industrial times (in W/m^2; right panel). The prior on radiative forcing in the top panel (b) is uniform (0.00, 2.10); the prior on radiative forcing in the other panels is triangular (0.00, 1.05, 2.10). The priors on climate sensitivity are from top to bottom uniform (0.00, 25.00); IPCC – triangular (0.14, 2.50, 6.22); expert 4 – Gumbel (2.27, 4.21); expert 5 – Gumbel (0.21, 0.16); expert 9 – Gumbel (2.09, 1.40); and Arrhenius – Normal (5.50, 7.56). Selected characteristics of the posteriors can be found in Table II.

Table II
Characteristics of six posterior distributions climate sensitivity and radiative forcing

Prior climate sensitivity	Climate sensitivity (°C)				Radiative forcing (W m^{-2})				BF[a]
	Mode	Median	Mean	Stdv	Mode	Median	Mean	Stdv	
Uniform[b]	4.2	8.5	10.2	6.3	0.35	0.71	0.85	0.53	–
Uniform[c]	5.1	7.5	9.0	5.4	1.05	0.86	0.89	0.41	–
IPCC[c]	3.5	3.6	3.6	1.1	1.18	1.25	1.27	0.35	0.24
Expert 4[c]	4.5	5.4	6.0	3.0	1.05	1.05	1.07	0.43	0.23
Expert 5[c]	0.3	0.3	0.4	0.2	1.05	1.15	1.15	0.36	0.01
Expert 9[c]	3.3	3.6	3.8	1.5	1.10	1.23	1.24	0.35	0.21
Arrhenius[c]	5.3	5.6	5.8	1.5	1.05	1.23	1.24	0.35	0.31
Composite[d]	3.8	4.4	4.9	2.4	–	–	–	–	–

[a] Bayes' factor, normalized to sum to unity.
[b] The prior for radiative forcing is uniform (0, 2.1).
[c] The prior for radiative forcing is triangular (0, 1.05, 2.1).
[d] The composite posterior on climate sensitivity, i.e., the sum of the five posteriors above, weighted by their Bayes' factor.

deviation of 0.2 °C reveals. The prior of expert number 5 is so sharp that it dominates the likelihood: Prior and posterior climate sensitivity are very similar; expert number 5 does not have to adjust his theory much to the data. The prior chance of a climate sensitivity greater than zero is 0.979; the posterior chance is 0.998. The IPCC (Houghton et al., 1994) is corrected though, for having estimated the radiative forcing a little bit too low. Here, most of the observed rise in the temperature is de facto explained by long term natural variability, as the enhanced greenhouse effect is a priori set to be even more unrealistic. See Figure 3 and Table II.

Expert number 9 is a bit boring. He represents the 'average' of the group of 14 experts, with a mean climate sensitivity of 2.9 °C and a standard deviation of 1.8 °C. Figure 3 and Table II reveal that the data are in favour of a climate sensitivity that is slightly higher, and a radiative forcing that allows for a thicker right tail. The prior chance of a climate sensitivity greater than zero is 0.997; the posterior chance is 1.000. As expert number 9 is probably a mainstream climatologist, we use this case for further illustration. Figure 4 depicts the bivariate prior of climate sensitivity and radiative forcing, the likelihood (degenerated on the curves $\zeta\omega = c$), and the posterior. The data attach a very low probability to the combination of low-forcing–low-sensitivity. The likelihood lies more towards the upper right corner than the prior does. Consequently, the posterior is shifted in that direction, and the prior is needed to discount high climate sensitivities.

The final prior is due to the great-grandfather of all climate change researchers, Svante Arrhenius (1896). The reason for incorporating his eminence is that Arrhenius did not have a change to examine the data, whereas the others had. So, his calculations are the only one guaranteed to be independent of the data; Arrhenius' prior is a real prior. As Arrhenius mentions very little on the uncertainty in his calculations, we have assumed a Normal prior, with Arrhenius' best guess of 5.5 °C, and a standard deviation of 2.75 °C (so that his estimate just deviates significantly from zero at the 5% level). Figure 3 and Table II show that the data strengthen this century old theory. The prior chance of a climate sensitivity greater than zero is 0.977; the posterior chance is 1.000.

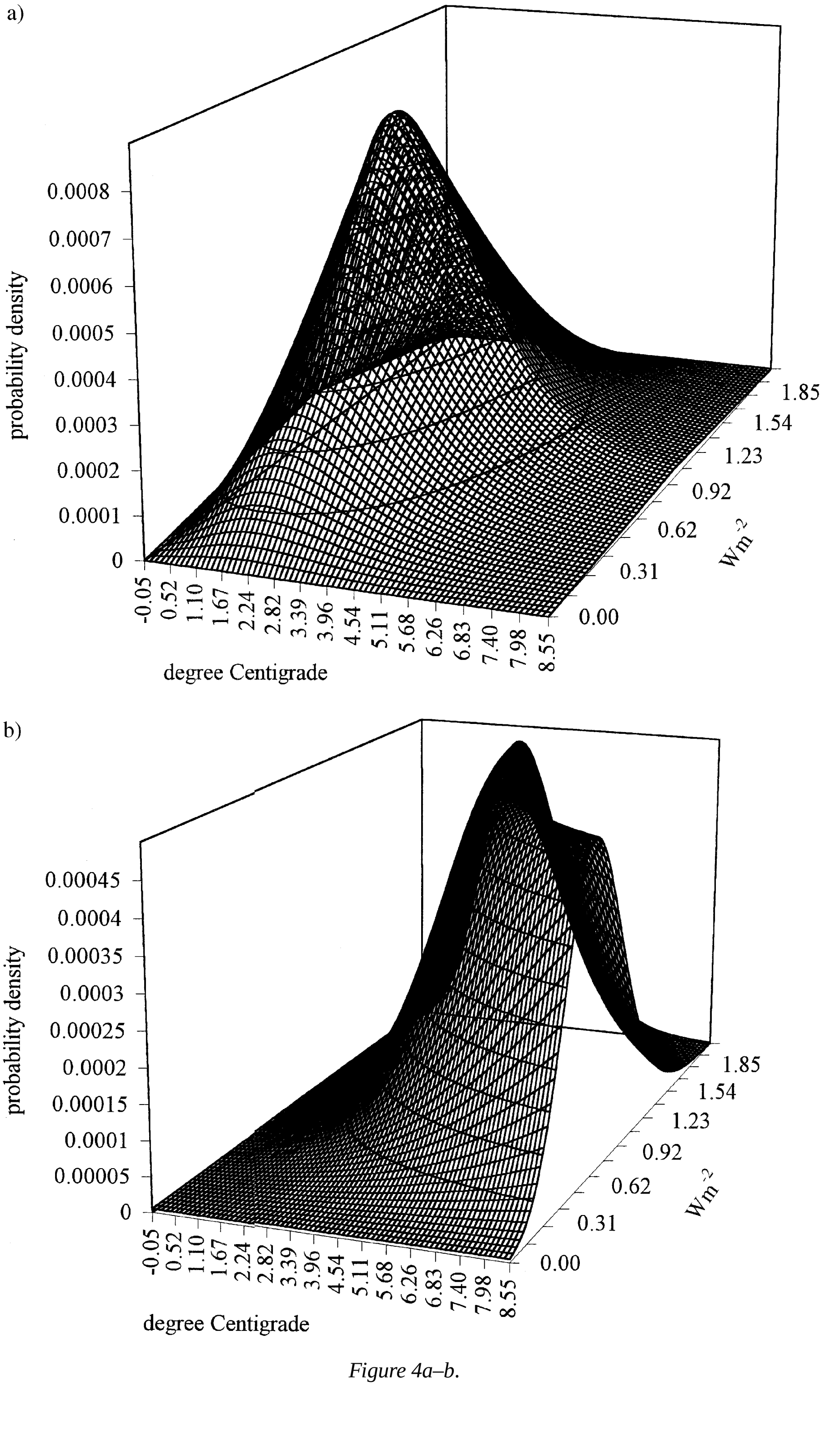
In sum, the answer obtained strongly depends on the prior chosen, a finding which is to a large extent the consequence of the lack of data on non-CO_2 forcing. This lack of knowledge is reflected in Equations (3) and (4) where non-CO_2 forcing is assumed to have run parallel to CO_2-forcing. We do demonstrate that a statistical analysis is possible, despite the identification problem of climate sensitivity and radiative forcing. Better data on the forcing of particularly sulphate aerosols, and corresponding model adjustments would be an improvement. However, two major points emerge from our primitive analysis. Firstly, a climate sensitivity of less than 0 °C is virtually excluded, independent of the choice of prior. The prior chances on low climate sensitivities are systematically discounted by the likelihood. The odd one out here is expert number 5, who is so certain of himself that empirical evidence will not shock his beliefs.

However, in a full Bayesian analysis, assuming that the five experts (counting the IPCC as an expert) a priori had equal chances of being right, the final probability distribution on climate sensitivity is a weighted average of the individual posteriors, with the *Bayes' factors* as weights. The Bayes' factor gives the probability that the expert is right, given the data (cf. Appendix B). The Bayes' factors are given in the last column of Table II. On a scale of one, to be divided over five experts, expert 5 scores only 0.01. Arrhenius scores best (0.31), followed by the IPCC. The final, composite posterior is depicted in Figure 5, together with its compounds. It should be noted that our selection of experts is subjective and not comprehensive, although we tried to make a representative selection. Different selections will result in slightly different final pictures.

The second major point emerging is that the posterior's right tail is rather fat, which has strong implications for decision making under uncertainty and risk aversion (Tol, 1995). Moreover, the fatness of the tail (in other words, the chance of a 'climate catastrophe') is determined by the prior. Particularly the tails of probability distributions based on expert elicitation are known to be unreliable (Morgan and Henrion, 1990). The implication of these two points is that research should focus on the high end of the climate sensitivity range rather than at the low end, and on non-CO_2 radiative forcing.

8. Conclusion

The enhanced greenhouse effect is a very good candidate to explain the observed global warming, as there exists a robust statistical relationship between the


a)
0.0008
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0.0006
0.0005
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0.0003
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probability density
-0.05 0.52 1.10 1.67 2.24 2.82 3.39 3.96 4.54 5.11 5.68 6.26 6.83 7.40 7.98 8.55
degree Centigrade
0.00 0.31 0.62 0.92 1.23 1.54 1.85
Wm-2
b)
0.00045
0.0004
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
0.00005
0
probability density
-0.05 0.52 1.10 1.67 2.24 2.82 3.39 3.96 4.54 5.11 5.68 6.26 6.83 7.40 7.98 8.55
degree Centigrade
0.00 0.31 0.62 0.92 1.23 1.54 1.85
Wm-2

Figure 4a–b.

c)

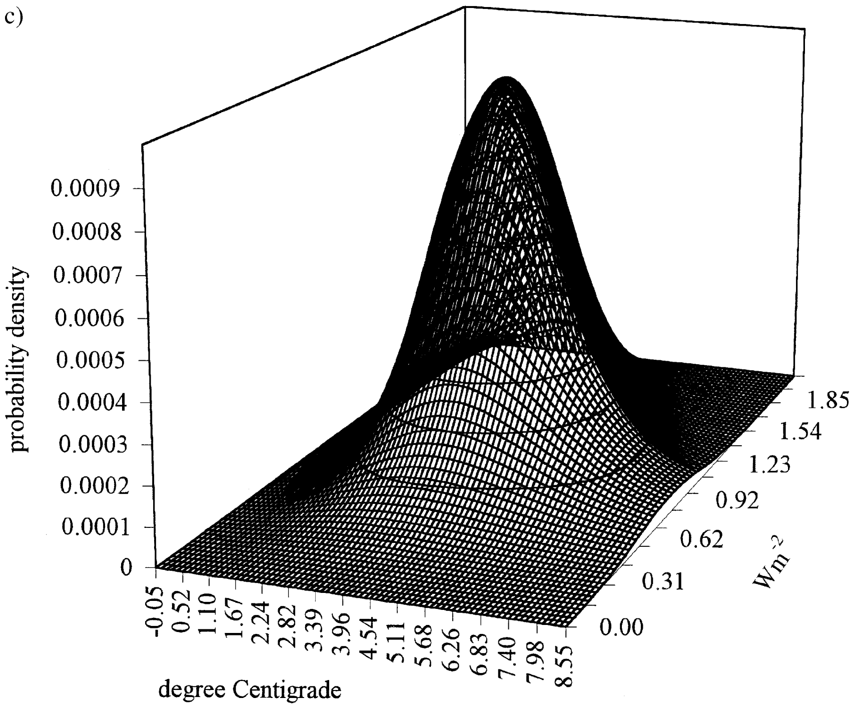

Figure 4c.

Figure 4. Bivariate prior (a), likelihood (b) and posterior (c) of the climate sensitivity to a doubling of atmospheric carbon dioxide equivalents and radiative forcing since pre-industrial times. The prior is the product of a triangular (0.00, 1.05, 2.10) distribution for radiative forcing and a Gumbel (2.09, 1.40) distribution (expert 9) for climate sensitivity. The marginals are depicted in Figure 3.

atmospheric concentration of carbon dioxide and the global mean surface air temperature. It is illustrated that the discussions on the short-term and the long-term dynamics can be separated, in line with co-integration theory. It is shown that a simple but adequate statistical model can describe a large part of the short-term natural variability reasonably well.

Detection of the influence of the enhanced greenhouse effect requires consideration of other explanations. Long-term natural variability is the most important one. It is shown that only if one has an extreme position on long-term natural variability, supported neither by the data, nor by the literature, the confidence in the relationship between carbon dioxide and temperature is affected. In the wide range of models considered, the odds are invariably in favour of the enhanced greenhouse effect as an explanation of the observed warming over the last century.

The emission of carbon dioxide is but one of the ways in which human beings potentially influence climate. Statistical analysis of these issues is hampered by the lack of data. We sketch a route along which the influence of other-than-CO_2 forcing, notable sulphate aerosols, can be included. Prior knowledge on the strength

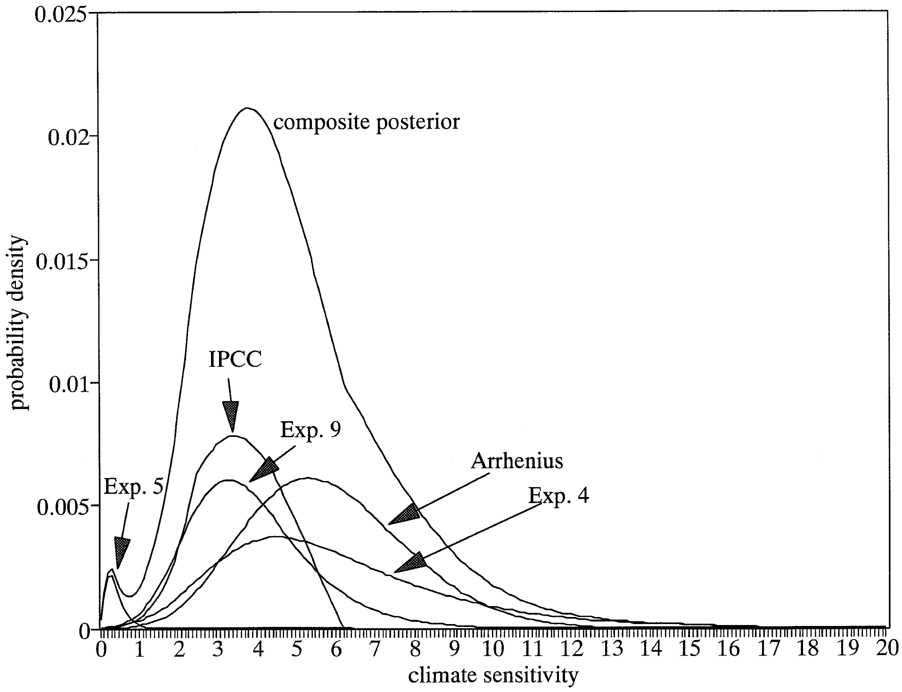

Figure 5. Composite posterior for the climate sensitivity, and its compounds. The composite posterior is the sum of the weighted posteriors of Figure 3 with the triangular prior on radiative forcing. The weights are the Bayes' factors of Table II.

of the relative forcings is of crucial importance here. We show how the information in the data can be combined with theoretical knowledge, and illustrate this by six cases. The data heavily discount the likelihood of a small climate sensitivity. The climate sensitivity may well be very large, if only the data are considered. Experts' knowledge suggests that the chance of a large climate sensitivity is much smaller.

Appendix A. Alternative Specifications of Models (1) and (1′)

A-1. STATIONARY ERROR STRUCTURES

A model like (1) may be considered as consisting of two parts: a causal CO_2 effect and an 'unexplained' part, modelled as a time series processes. We denote the unexplained part as natural variability of the global mean temperature. Different specification for the natural variability may lead to rather different conclusions for the CO_2 effect, because some time series processes, particularly (near) nonstationary ones can 'replace' the enhanced greenhouse effect as an explanation of the observed warming. To explore this, different time series models are applied to the

Table A-I
Model (1) with alternative error structures

Model[a]	ln[CO2][b]	Stdv[c]	σ[d]	*LL*[e]	*LL*[f]
ARMA(0,0)	5.0048	0.3018	0.1138	93.0048	20.3279
ARMA(1,0)	5.0460	0.4649	0.1038	104.7171	87.9799
ARMA(2,0)	5.0132	0.4084	0.1032	105.9698	88.6203
ARMA(3,0)	5.0346	0.4414	0.1033	106.3695	93.2976
ARMA(4,0)	5.0922	0.4969	0.1029	107.4486	97.7703
ARMA(0,1)	5.0048	0.2741	0.1034	105.2473	56.7897
ARMA(1,1)	5.0048	0.3298	0.1032	105.9799	89.6097
ARMA(2,1)	5.0048	0.3655	0.1036	106.0997	94.0444
ARMA(3,1)	5.0864	1.1172	0.1035	106.6943	99.6908
ARMA(4,1)	5.0228	0.4231	0.1033	107.4918	99.2754

[a] Error structure for model (1).
[b] Estimated influence of $\ln[CO_2]_{t-20}$ on global mean temperature.
[c] Standard deviation of parameter estimate.
[d] Standard error of regression.
[e] Loglikelihood.
[f] Loglikelihood if carbon dioxide is excluded from the model and the other parameters are re-estimated.

global mean temperature record. The last column of Table A-I gives the result.* It appears that one needs a more richly parameterized model than AR(1) to get a reasonable description of the data. An ARMA(3,1) process comes closest in performance to model (1). Nevertheless, its loglikelihood is still about 6 points lower than the same model including carbon dioxide. This is strong evidence according to all standards: the Classical χ^2 test requires more than 1.9 points (5% significance level) and the Bayesian Schwarz criterion requires more than 2.4 points to justify the inclusion of carbon dioxide.

The time series models that really affect the conclusions on the influence of the enhanced greenhouse effect are those with a (near) unit root (see section A-3). In Table A-I, the ARMA(3,1) specification is an example – the standard deviation of the $\ln[CO_2]$ coefficient is much larger than in the other cases.

A-2. DETERMINISTIC TRENDS

Table A-II shows that the result of model (1′) is not restricted to the assumption of linear long-term natural variability. The two top rows repeat model (1) and (1′), respectively. The two bottom rows add a quadratic and cubic trend to the linear trend. The estimated influence of carbon dioxide on the temperature increase, but remains significantly different from zero. Table A-II also displays the estimated natural variability, that is, the temperature change over the period 1870–1991. The

* Full model output is available from the authors on request.

Table A-II
Model (1′) with various specifications of natural variability

Model[a]	$\ln[CO_2]$[b]	Stdv[c]	σ[d]	LL[e]	Nat. var.[f]
None	5.0460	0.4649	0.1038	104.7171	0[g]
Linear	6.1398	2.1990	0.1042	104.8479	–0.1328
Quadratic	7.1514	3.7102	0.1046	104.9067	–0.2490
Cubic	9.2540	3.8572	0.1047	105.3352	–0.5784

[a] Maximum power natural forcing.
[b] Estimated influence of $\ln[CO_2]_{t-20}$ on global mean temperature.
[c] Standard deviation of parameter estimate.
[d] Standard error of regression.
[e] Loglikelihood.
[f] Estimated natural temperature change over sample period.
[g] By assumption.

estimated temperature change is negative in all cases, and large compared to the priors on natural variability used above.

A-3. NONSTATIONARY ERROR STRUCTURES

Table A-II only considers deterministic long-term natural variability, and Table A-I only looks at stationary (hence, short-term) natural variability. Table A-III adds nonstationary (hence, long-term) natural variability. Nonstationary long-term natural variability is the most serious competitor for the enhanced greenhouse effect as an explanation for the observed warming. In this specification, the long-term variance of the temperature is infinite; on the short run, nonstationary models can show trending, even accelerating behaviour, although the chance of this is small. Note that ‘long-term memory’ (or fractional integration) error processes lie somewhere between stationary and nonstationary errors (cf. Granger and Joyeux, 1980). The top row repeats the findings of model (1′). In the second row, the AR parameter is forced to one (from its maximum likelihood estimate of 0.41). The estimated influence of carbon dioxide falls considerably and is highly insignificant. However, this model performs substantially worse than (1′). In the third row, an ARMA(3), error structure is used, with the first AR root forced to lie on the unit circle (ARIMA(2,1,1)). The estimated climate sensitivity is closer to its original estimate but remains insignificant; model performance is still below (1′) despite the fact that three additional parameters are used. The bottom row of Table A-III restricts carbon dioxide to be zero. The message of Table A-III is that, although it is possible to explain the observed warming with a nonstationary model without carbon dioxide, this description is outperformed by models that do contain carbon dioxide. In addition the estimated natural trend is large whether carbon dioxide is included (at least 0.11 °C/century) or excluded (0.52 °C/century). According to the ARIMA(2,1,1) model without carbon dioxide, the standard deviation of the global

Table A-III
Alternatives to model (1′)

Model	$\ln[CO_2]$[a]	Stdv[b]	t[c]	Stdv[d]	σ[e]	*LL*[f]
(1′)	6.1398	2.1990	0.1098	0.2161	0.1042	104.8479
$\phi = 1$[g]	2.6401	7.2519	0.1693	1.3503	0.1231	83.4315
ARIMA(2,1,1)[h]	4.4526	6.7405	0.1488	1.6671	0.1066	102.5104
ARIMA(2,1,1)[h]	0[i]	–	0.5229	1.6471	0.1080	100.4105

[a] Estimated influence of $\ln[CO_2]_{t-20}$ on global mean temperature.
[b] Standard deviation of parameter estimate carbon dioxide.
[c] Estimated slope of linear trend.
[d] Standard deviation of parameter estimate linear trend.
[e] Standard error of regression.
[f] Loglikelihood.
[g] Model (1′) with the AR parameter forced to one.
[h] Model (1′) with an ARIMA(2,1,1) error structure.
[i] By assumption.

mean temperature over a period of 100 years is a reasonable 0.13 °C; the explanation is that the best estimate for the MA parameter is 0.98 (in both ARIMA(2,1,1) models), thus eliminating most of the nonstationarity in the AR process.

Appendix B. Some Technical Issues

This appendix briefly presents four techniques used in the main text, with which not every reader is familiar. These techniques are mixed estimation, Almon transformation, marginalization, and Bayes' factors.

B-1. MIXED ESTIMATION

The Classical procedure of mixed estimation (Theil and Goldberger, 1962) is computationally equivalent to the Bayesian procedure of a conjugate, partly non-informative prior. Let κ denote the vector of estimates of the parameters of a linear regression model, and let Σ denote its covariance matrix. Suppose that the uncertainty around κ is (approximately) Normal. Let us assume that we had strong theoretical ideas about the parameters, described by a vector of alternative estimates λ. Suppose that our uncertainty about λ is described by a Normal distribution, with covariance matrix T. The combined estimate μ then takes the form

$$\mu = (\Sigma^{-1} + T^{-1})^{-1}(\Sigma^{-1}\kappa + T^{-1}\lambda) \tag{B-1a}$$

with covariance

$$Y = (\Sigma^{-1} + T^{-1})^{-1}. \tag{B-1b}$$

This is the mixed estimator, or the Bayesian posterior. μ is a weighted average of κ and λ, the weights being the respective inverse covariance matrices (also known as precision matrices). Commonly, T is a diagonal matrix. Those elements for which we have no a priori information available have an infinite variance in T, and hence the corresponding element in λ has a zero weight in μ through T^{-1}. Thus, if the prior is completely non-informative, $\mu = \kappa$ and $Y = \Sigma$. In the above, only information on the long-term natural variability is used.

B-2. ALMON TRANSFORMATION

Suppose that Y depends not only on contemporaneous values of X, but also on values of X in the long past and everything in between. Then, the regression equation may look like

$$Y_t = c + \sum_{i=0}^{s} \gamma_i X_{t-i} + \varepsilon_t \,. \tag{B-2}$$

If s is large (above, $s = 40$), many parameters γ need to be estimated, for which data may not be sufficient. In addition, estimations of γ may well show an irregular pattern which, for theoretical reasons, could be considered undesirable. If X shows regular behaviour, lagged values of it are multicollinear. Almon (1962) proposed a solution, namely to constrain γ to lie on a polynomial. In our case, this polynomial is of the second order, that is,

$$\gamma_i = \begin{cases} \alpha_0 + \alpha_1 i + \alpha_2 i^2 & \text{for } i = 1, 2, \ldots, s \\ 0 & \text{otherwise.} \end{cases} \tag{B-3}$$

Thus, only three values of α, in lieu of s γ, need to be estimated. Moreover, we constrained our endpoints to be zero, that is, $\gamma_{-1} = \gamma_{s+1} = 0$, so that only one α remains. In the estimation algorithm, the X record (atmospheric carbon dioxide) is replaced by a record of appropriately weighted past values of X, so that estimation of α is straightforward. γ follows by implication.

B-3. MARGINALIZATION

The posterior distribution of ζ, ω and β follows from Bayes' rule,

$$p(\zeta,\ \omega,\ \beta|\text{data}) \propto \pi(\zeta,\ \omega,\ \beta) l(\text{data}\,|\zeta\omega,\ \beta). \tag{B-4a}$$

However, the interest is in $p(\zeta,\ \omega|\text{data})$, the posterior distribution of climate sensitivity and radiative forcing. By the rule of total probability,

$$p(\zeta,\ \omega|\text{data}) = \int_B p(\zeta,\ \omega,\ \beta|\text{data})\ \mathrm{d}\beta \,. \tag{B-4b}$$

Then, by Bayes' rule and by assuming the prior on β independent of ζ and ω,

$$\int_B p(\zeta,\ \omega,\ \beta|\text{data})\ \mathrm{d}\beta \propto \int_B \pi(\zeta,\ \omega)\pi(\beta) L(\text{data}\,|\zeta\omega,\ \beta)\ \mathrm{d}\beta \,. \tag{B-4c}$$

Simple manipulation yields

$$\int_B \pi(\zeta,\ \omega)\pi(\beta)L(\text{data}\,|\zeta\omega,\ \beta)\ \text{d}\beta = \pi(\zeta,\ \omega)\int_B \pi(\beta|\zeta\omega)L(\text{data}\,|\zeta\omega,\ \beta)\ \text{d}\beta\,. \tag{B-4d}$$

Marginalizing the likelihood on β, using the prior on the long-term natural variability, results in

$$\pi(\zeta,\ \omega)\int_B \pi(\beta|\zeta\omega)L(\text{data}\,|\zeta\omega,\ \beta)\ \text{d}\beta = \pi(\zeta,\ \omega)L_{|\beta}(\text{data}\,|\zeta\omega)\,. \tag{B-4e}$$

So, we can proceed with our analysis without worrying about natural variability.

The same procedure can be used to marginalize on ω:

$$\begin{aligned} p(\zeta|\text{data}) &= \int_\Omega p(\zeta,\ \omega|\text{data})\ \text{d}\omega \propto \int_\Omega \pi(\zeta,\ \omega)L_{|\beta}(\text{data}\,|\zeta\omega)\ \text{d}\omega \\ &= \pi(\zeta)\int_\Omega \pi(\omega)L_{|\beta}(\text{data}\,|\zeta\omega)\ \text{d}\omega = \pi(\zeta)L_{|\beta,\omega}(\text{data}\,|\zeta)\,. \end{aligned} \tag{B-5}$$

B-4. BAYES' FACTOR

The integral over β over the last part of (B-5),

$$BF = \int_Z \pi(\zeta)L_{|\beta,\omega}(\text{data}\,|\omega)\ \text{d}\zeta\,, \tag{B-6}$$

is known as the Bayes' factor. It is the area under the unscaled posterior density and, as such, a measure of the correspondence between prior and likelihood. Note that the Bayes' factor is a relative measure, that is, it can only be used to compare two or more different priors to one another.

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